

## E101. Study of the resonance in RLC circuits

### 1. Introduction.

An RLC circuit is an electrical circuit consisting of a resistor, an inductor, and a capacitor, connected in series or in parallel. The RLC part of the name is due to those letters being the usual electrical symbols for resistance, inductance and capacitance, respectively. The circuit forms a harmonic oscillator for current and will resonate in a similar way as an LC circuit will. The main difference that the presence of the resistor makes is that any oscillation induced in the circuit will die away over time if it is not kept going by a source. This effect of the resistor is called damping.

In alternating current (AC) circuits of angular frequency  $\omega$  ( $\omega=2\pi f$ ,  $f$ - frequency) the relation between voltage ( $U$ ) and current ( $I$ ) is given by:

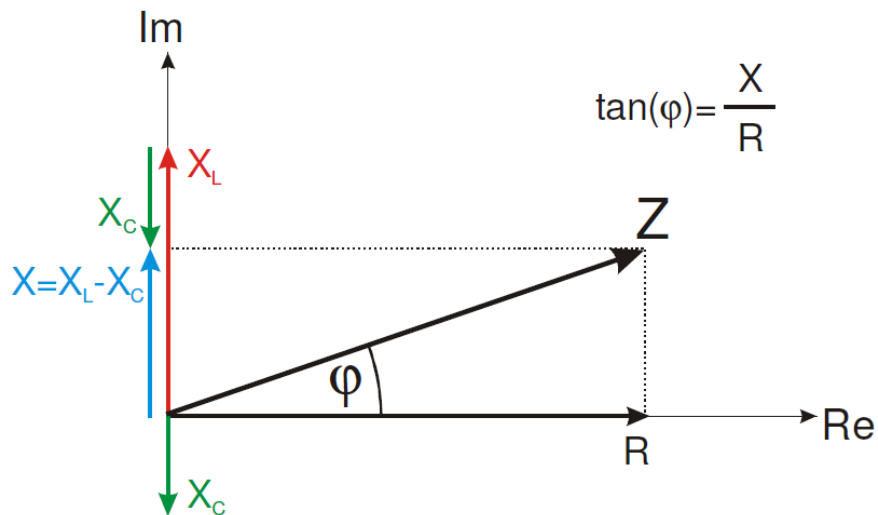
$$\frac{U}{Ie^{-j\varphi}} = z = R + j\omega L + \frac{1}{j\omega C} \quad (1)$$

where  $z$  is called impedance and its real part is the resistance ( $R$ ), while its imaginary parts are connected with inductance ( $L$ ) and capacitance ( $C$ ). The ratio of imaginary and real part of impedance defines the tangent of the angle of the phase shift between the voltage and current ( $\varphi$ ), see Fig. 1.

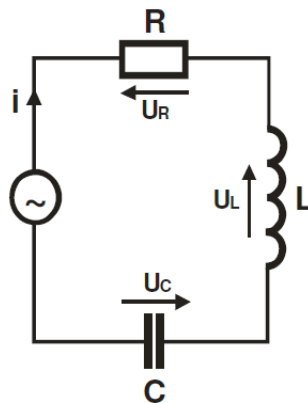
In the series RLC circuit (Fig. 2) the ratio of the voltage on the resistance  $U_R$  over the applied voltage  $U_0$  can be calculated as:

$$\frac{U_R}{U_0} = \frac{IR}{IZ} = \frac{R_p}{\sqrt{(R_p + R_L)^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}} \quad (2)$$

where  $R_p$  is the resistance of the resistor, while  $R_L$  is the resistance of the inductor (electrical coil).



**Figure 1.** Real and imaginary part of impedance.



**Figure 2.** RLC series circuit.

Similarly, the angle of the phase shift between the input and output signal (total voltage and current on the resistor),  $\varphi$ , is given by:

$$\varphi = \arctan\left(\frac{\omega^2 LC - 1}{\omega C(R_p + R_L)}\right) \quad (3)$$

Resonance occurs because energy is stored in two different ways: in an electric field as the capacitor is charged and in a magnetic field as current flows through the inductor. The resonance of the voltage occur for the characteristic frequency  $f_0$ :

$$f_0 = \frac{1}{2\pi\sqrt{LC}} \quad (4)$$

For the resonance frequency, the imaginary part of the impedance is equal zero (the contribution of capacitance and inductance cancel each other), so the phase shift  $\varphi$  is equal zero and the voltage ratio  $U_R$  over  $U_0$  reaches maximum value (see eqs. (2) and (3)).

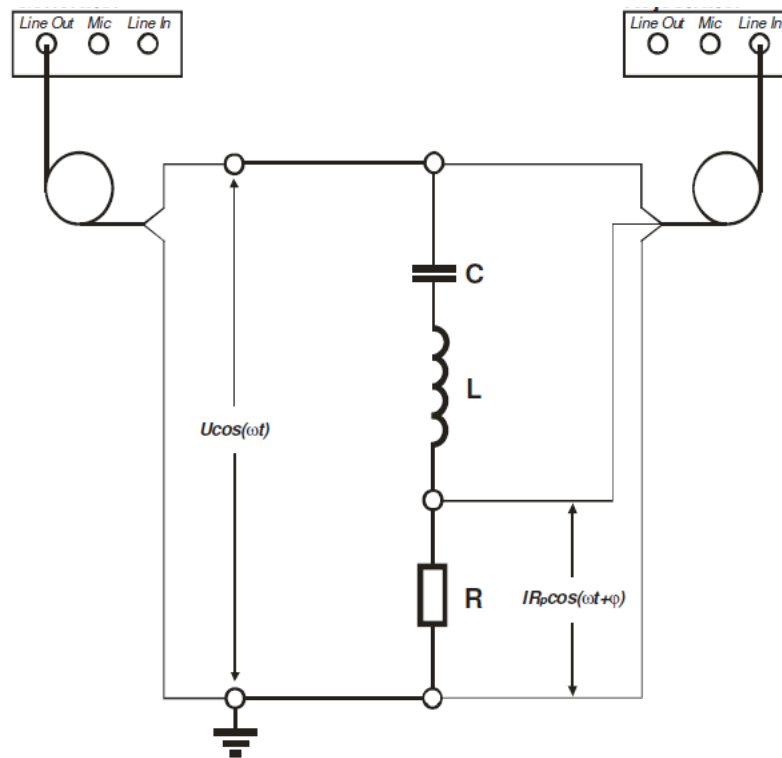
The bandwidth  $\Delta f$  of the resonance peak (with maximum at resonance frequency  $f_0$ ) is measured between the 3dB-points, that is, the frequencies at which the power passed through the circuit has fallen to half the value passed at resonance. There are two of these half-power frequencies, one above, and one below the resonance frequency:  $\Delta f = f_2 - f_1$ , where  $f_1$  is the lower half-power frequency and  $f_2$  is the upper half-power frequency. The Q-factor is a widespread measure used to characterize resonators. It is defined as the peak energy stored in the circuit divided by the average energy dissipated in it per cycle at resonance. Low Q circuits are therefore damped and lossy and high Q circuits are underdamped. The Q factor is defined as:  $Q = f_0 / \Delta f$ . For the a series resonant RLC circuit it can be also expressed as:

$$Q = \frac{1}{R_p + R_L} \sqrt{\frac{L}{C}}. \quad (5)$$

## 2. Apparatus and measurements.

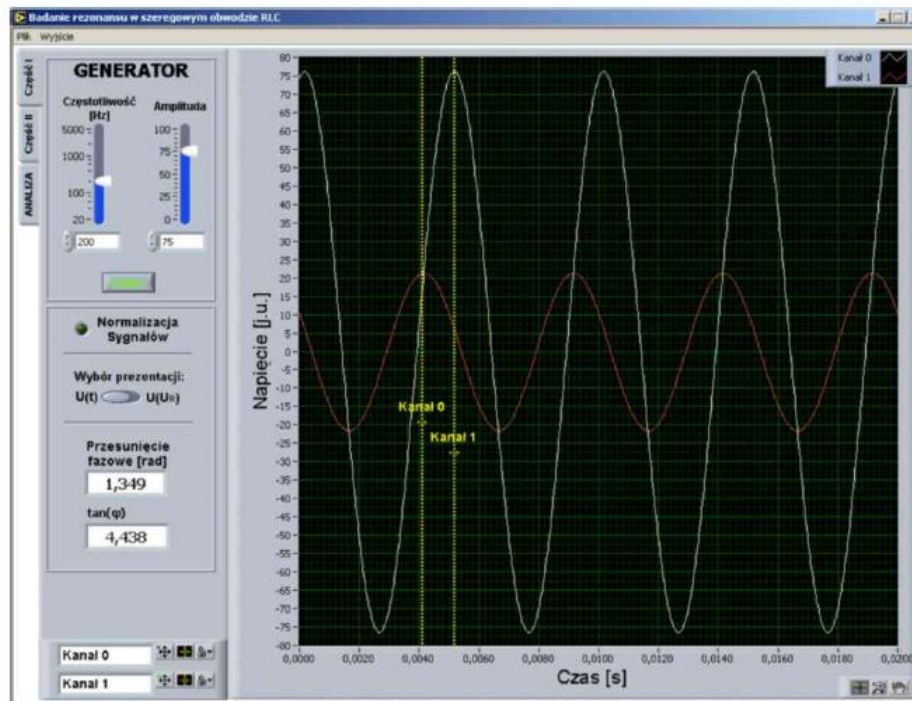
Two sound cards installed on the computer are used for generation and registration of the signals. Each standard sound card has two digital-to-analog converters (one for each channel) serving for signal generation, and two analog-to-digital converters serving for signal registration. In the case of typical sound cards the signal is sampled at a frequency of 44.1 kHz and 16 bits of accuracy. These are parameters with sufficient precision to perform measurements of frequency characteristics in the range of 20 - 20000 Hz.

We combine the system according to the diagram shown in Figure 3. The system consists of connected in series: resistor R (of resistance  $R_p$ ), coil L (of resistance  $R_L$ ) and a capacitor C. Source of alternating voltage supplied to the system under study is one of the two *Line Out* channel of the first sound card. Both channels *Line In* of the second card are used to record the voltage at the terminals of the source and the voltage drop across the resistor R.



**Figure 3.** Scheme of the connections used in the measurements.

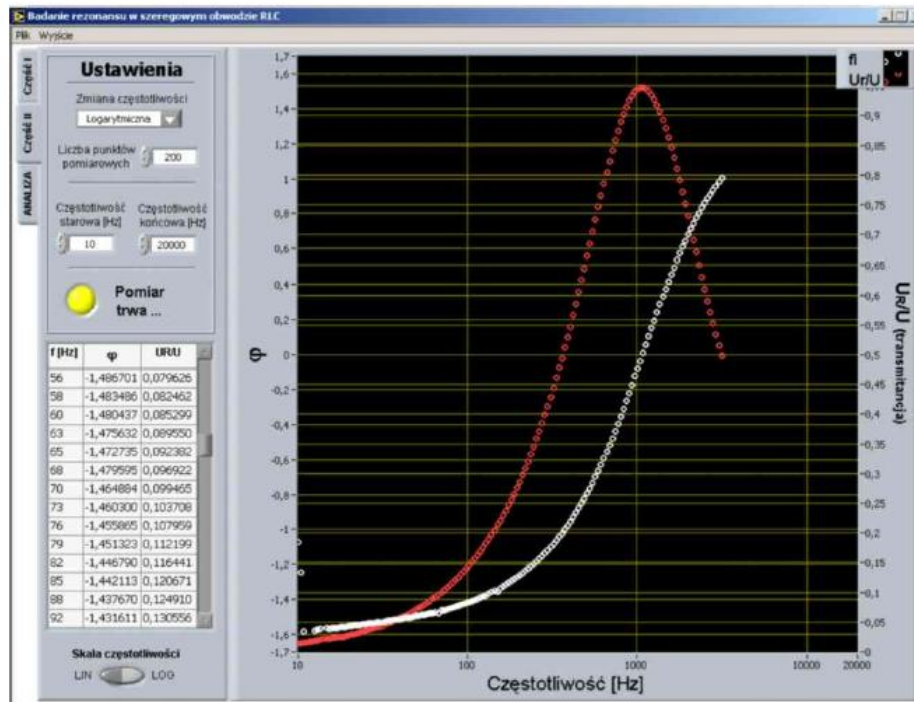
The measurements are held in two stages. The first part is to observe the changes in parameter values of the serial RLC (phases differences between current and applied voltage and the voltage drop across the resistor) induced by the changes in the frequency of the supply voltage, see Figure 4. Button *START* launches the generator of the sinusoidal signal, which frequency and amplitude can be varied by the sliders *Częstotliwość* and *Amplituda*, respectively. Signals registered in both channels (*Kanał 1* and *Kanał 2*) are presented on the plot. The variation of the amplitude and phase shift can be observed. Using cursors visible on the chart, one can specify on the variations in the position of the maxima of the two signals. Difference is presented in the form of the value of the phase shift angle (*Przesunięcie fazowe*) and its tangent. It might be convenient to use the option of the signal normalization (*Normalizacja sygnałów*). Option *Wybór prezentacji* enables to switch between the presentation in the mode  $U(t)$ ,  $U_R(t)$  and the mode  $U(U_R)$ , the latter being the plot of so called Lissajous curve.



**Figure 4.** The panel of the application (*RLC.exe*) used for the first part of the laboratory.

The task consists in registering voltage waveforms for several selected frequencies. It should be noted that for frequency lower than the resonance frequency the phase of current goes ahead of the voltage phase ( $\varphi \rightarrow -\pi/2$ ) while for  $f > f_0$  the current phase is delayed relative to the voltage ( $\varphi \rightarrow +\pi/2$ ). When the results are presented in the form of Lissajous curves, one should easily find the approximate resonant frequency  $f_0$  based on characteristic changes in the shape of a loop.

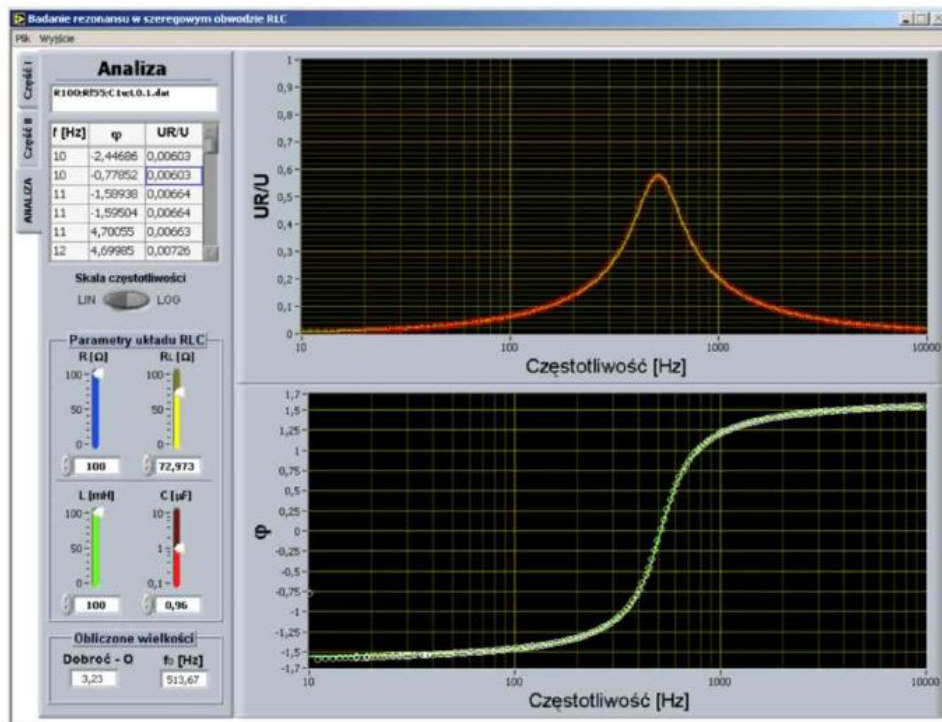
In the second part of the laboratory the frequency characteristics of the transmittance and the phase shift of the circuit will be measured and the impact of changes in the value of the resistance  $R$ , the inductance  $L$  and the capacitance  $C$  on the  $Q$ -factor and resonance frequency will be measured. In the second fold of the application *RLC.exe* (called *Część II*, see Figure 5) one has to give the start and stop frequency (*Częstotliwość początkowa* and *Częstotliwość końcowa*), select the number of points to be measured (*Liczba punktów pomiarowych*) and decide whether the frequencies will be scanned in a linear (*Liniowa*) or logarithmic (*Logarytmiczna*) way. Then, *Start* button should be pressed, and when the measurements are finished, the data should be saved to the file (menu *Plik*, option *Zapis*).



**Figure 5.** The panel of the application (*RLC.exe*) used for the second part of the laboratory.

Next, the third fold of the application *RLC.exe* (called *Analiza*, see Figure 6) is used to analyze the signals. The data should be loaded from the file (menu *Plik*, option *Odczyt*) and the correct values of  $R_p$ ,  $R_L$ ,  $L$  and  $C$  should be found for which the fit of the theoretical curves (calculated from eq. (2) and eq. (3)) to the experimental data is the best. The values of Q-factor and resonance frequency  $f_0$  (calculated from eq. (4) and eq. (5)) should be noted (lower-left frame in Figure 6).

This procedure should be repeated for several RLC circuits in which the resistor, capacitor and inductor are changed. The influence of the R, L and C parameters on the shape of the resonance curve, resonance frequency and Q-factor should be discussed.



**Figure 6.** The panel of the application (*RLC.exe*) used for the analysis of the frequency characteristics of  $U_R/U$  and  $\varphi$ .