M103. Atwood machine for investigation of uniformly accelerated motion

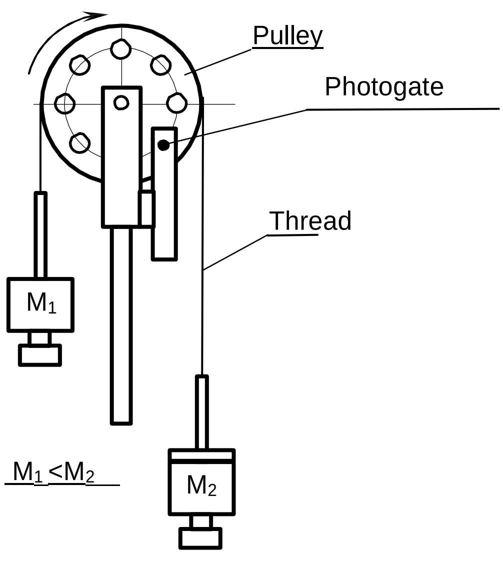
Aim: *Measurement of instantaneous values of position, velocity and acceleration in uniformly accelerated motion in a system made of a pair of weights hanging on a pulley. Confirmation of the Newton's second law of dynamics.Determination of gravitation constant g.*

Problems:
Uniformly accelerated motion (path, velocity, acceleration as a function of time).
Newton's second law of dynamics.
Moment of force, angular velocity, angular acceleration, moment of inertia.

Literature*:*
David Halliday, Robert Resnick, Jearl Walker "Podstawyfizyki" Wyd.Nauk.PWN,Warszawa 2003

1. Introduction

Atwood machine is an instrument for verification of the laws governing the uniformly accelerated motion. It is built of a pulley over which two weight of different mass are hanged with the use of a thread. In quantitative description of Newton's laws of dynamics the most disturbing is friction. In Atwood machine the effect of friction has been minimised and reduced to the resistance of the pulley bearing, see Fig. 1.



Description of the system needs determination of all forces and moments of forces and derivation of equations of motion for both masses and the pulley. We assume that the thread is massless and inextensible. The paths covered by the two weights, their velocities and accelerations are the same. Each mass is subjected to two forces of the opposite directions, the gravitational force,*gM*,and the force of the thread tension *N*. As all forces act in the same direction, it is sufficient to consider only the vertical direction. Let's assume that the acceleration of a weight moving down is positive, while that of the weight moving up is negative. As we assume that $M_2>M_1$, weight 2 moves down. The resultant forces acting on masses M_1 andM_2 are:

$$\begin{aligned} F_1 &= -aM_1 = gM_1 - N_1 \\ F_2 &= +aM_2 = gM_2 - N_2 \end{aligned} \tag{1}$$

Also the pulley is set in motion and its inertness has to be considered. According to the Newton's second law of dynamics, the rotational motion of a pulley of moment of inertia *I* with the angular acceleration ε, is induced by the action of a constant moment of force$L$. The origin of the moment of force is the force of tension of the thread N_1andN_2,applied on the two sides of the pulley at a distance *r*from its axis. The resultant moment of force is a sum of the two moments L_1andL_2. The rolling friction of the pulley bearing, responsible for the moment of the friction force L_T, slowing down the rotation, should also be taken into account. The equation of motion for the rotational motion can be written as

$$L = L_2 - L_1 = I\varepsilon + L_T, \tag{2}$$

$$N_2 r - N_1 r = I\varepsilon + F_T r. \tag{3}$$

So assuming the lack of sliding friction, the angular acceleration is $\varepsilon = a/r$, where*a* is the linear acceleration of the motion of the weights.

Assuming that the pulley is a cylinder of mass *m*, its moment of inertia is I=1/2mr^2. Substituting eqs. (1) into eq. (3), the final equation of motion of the entire system takes the form:

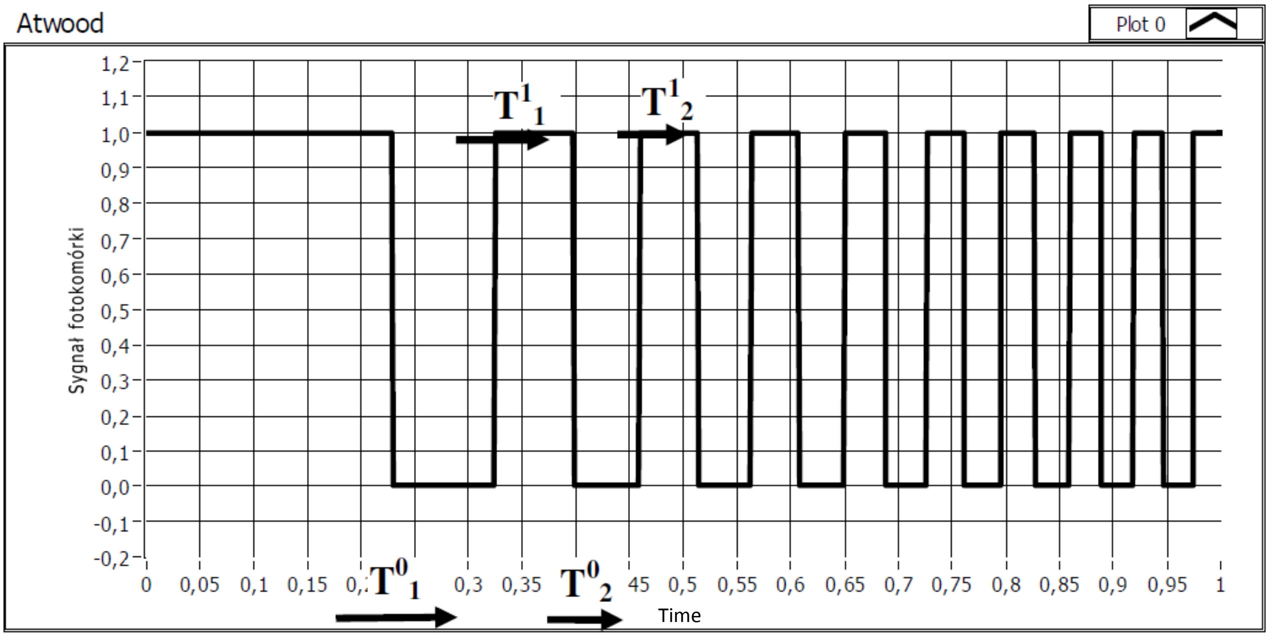
$$(M_2 - M_1)g = (M_1 + M_2)a + \frac{1}{2}ma + F_T. \tag{4}$$

$$a = \frac{M_2 - M_1}{M_1 + M_2 + \frac{1}{2}m} g - F_T. \tag{5}$$

This equation is written disregarding the effects related to the pulley shape irregularities (hollows and protuberances) and the presence of a furrow on the pulley which makes the radius in the equation for the moment of inertia differ from that from which the force acting on the thread is calculated.

The measuring setup is constructed so that a photocell records the light from photodiode passing through the holes in the pulley. In the plot of the time dependence of the photocell signal, the sections between the rapid increase and rapid decrease of the photocell signal correspond to the passages of the holes over the photocell. Assuming that the acceleration is not very high, the change in velocity between the passages of neighbouring holes is small, which means that the time dependence of velocity so also the time dependence of path are rather smooth.

2. Instantaneous values of velocity, acceleration and path in uniformly accelerated motion



Exemplary time dependence of the photocell signal.

The above figure shows exemplary time changes in the photocell signal. As the size of the holes is not equal to the distance between the edges of subsequent holes, the elementary section of the path is the distance d between the same (right or left) edges of the holes or their centres, so the sum of the lengths of the upper and lower arrows. The distance between the holes and the axis of rotation is smaller than the distance between the thread and the axis, so it is convenient to consider the angular velocity$\omega = d\alpha/dt$,where $\alpha$is an instantaneous angular position of the pulley. The linear velocity at an arbitrary distance R from the axis of rotation is found as the product ωR. The distance between the holes d can be replaced by a constant angle $\beta = 2\pi/N$, where$N$ is the number of holes. If T_i^1 is the time between passages of two subsequent right edges of the holes (upper arrows) and T_i^0 is the time between passages of two subsequent left edges of the holes (lower arrows), the instantaneous velocity in the *i*-th cycle is:

$$\omega_i = \frac{\beta}{T_i^1 + T_i^0}, \qquad v_i = \omega_i r \,, \tag{6}$$

where$v_i$is the linear velocity of the thread. The difference in the lengths of subsequent arrow is a consequence of the fact that they illustrate the time of covering the same path sections with increasing velocity. The time moments t_i,at which the linear velocity of the thread is v_i correspond to the half of the sum of the arrows T_i:

$$t_i = t_{i-1} + \frac{1}{2}(T_i^1 + T_i^0), \,, \tag{7}$$

initially we assume t_0= 0. Knowing the values of $v_i$and t_i it is possible to calculate the instantaneous values of acceleration $a_i$as the values of appropriate difference quotients. However, it has sense only if $t_0$is really equal to zero, while in a particular series of measurements its value is random, depending on the time between the release of the weights and the start of measurement.

An alternative method for determination of the average value of acceleration is to find it from the slope of the dependence *v*(*t*) by linear regression, see the box Classical regression in UserLibrariesin the palette of functions. The value of parameter obtained from linear regression (the slope)is a measure of acceleration, while the value of parameter b in the equation of linear regression (the point of intersection with the *y* axis *Y*) permits determination of the real moment of starting the motion in the time scale assumed for the calculation of t_i.The *v*(*t*) plot shifted by this value should pass through the centre of the coordination system.

Determination of the path covered by the pulley on time is much simpler as it is equal to the multiple of 1/*N* of the pulley circumference (*N* – the number of holes in the pulley):

$$s(t_i) = s_0 + i\,\frac{2\pi r}{N}\,. \tag{8}$$

This can be compared to the classical equation for distance passed in linear accelerated motion:

$$s(t) = s_0 + \frac{at^2}{2}\,. \tag{9}$$

The comparison can be made e.g. by analysis of the slope of $s(t^2)$ dependence, making use of the box Classical Regression.

The two acceleration values determined, a, should be compared with the value calculated from eq. (5) on the basis of the directly measured values of M_1, M_2, r and a given value of m and g taken from the tables.

The above procedure should be repeated for different combinations of masses M_1 and M_2 in two series. In the first series measurements of acceleration should be performed as a function of the mass difference $\Delta M = M_1 - M_2$ at the fixed sum of masses M_1+M_2 = const, while the second series should be performed at the fixed ΔM. The obtained dependencies of the acceleration on masses of the weights $a(\Delta M)$ and $a(M_1 + M_2)$ can be used for determination of the gravitational constant g, by the method of linear regression. For the second series of measurements the problem should be linearized by substituting $x=1/(M_1+M_2+m/2)$.

No motion is friction free. As the masses of the weights used are small, the moment of friction slows down the movement of the pulley to such an extent that it should be taken into account in the calculation of the gravitational constant. It is recommended to start from determination of the resistance force acting on the pulley under different loading of the thread. To do this, you should hang weights of the same mass on both hooks and set them in motion with your hand, then quickly start the measurement to record the moment at which their velocity decreases. The measured (negative) acceleration a_T is a measure of the friction force. To determine correctly the gravitational constant g, in eq.(5) the parameter a should be replaced by $a + |a_T|$. As the friction force F_T increases with increasing loading of the pulley and it is nonzero under no loading, (F_0), you can check the character of this relation.

Assuming
$$F_T = F_0 + kM, \tag{10}$$
and writing
$$F_T = a_T M, \tag{11}$$
we get
$$a_T = F_0 / M + k, \tag{12}$$

where k is the coefficient of proportionality between the total mass M loading the pulley and the force of friction. Measurements of a_T for a few values of M should bring a linear dependence $a_T(1/M)$.

The measuring instrument used in this experiment includes a programmable microprocessor capable of realisation of complex tasks with the use of analog-digital transducers, digital input and output and counters. Communication with the computer is realised through a series connection and the list of commands for the microprocessor and parameters of transmission are given in Additional Information.

The device permits measurements of subsequent high and low states of the photocell.

In this version of the experiment, students can use a program realising the entire procedure, starting from the set of work parameters to data collection and their preliminary analysis.

Tasks*:*

1. *Measure the diameter of the pulley. Choose a few weights, measure their masses and check if they are in agreement with the nominal value. The mass of the pulley, hooks and thread is given in the appropriate windows of the program.*
2. *Modify eqs. (3-5) to take into account the furrow on the pulley's edge. Measure its depth and estimate the difference between the acceleration values calculated from the unmodified and modified equation (5).*
3. *Using the program for determination of the instantaneous values of velocity (v), acceleration (a),and the friction force (T), determine the relation between the friction force and the total mass of weights and the velocity of their motion. Perform the measurements for the weights of the same masses.*
4. *Perform a few measurements of acceleration filling the field "expected friction" with the value corresponding to the conditions of the experiment. Compare the values of the gravitational constant g calculated without the correction for friction and with this correction taken into regard.*
5. *Assuming that the friction does not depend on the weights velocity and masses, the values of friction and gravitational constant g can be obtained from the dependence of acceleration on the total mass of the weights (or the force inducing this acceleration). Determine the acceleration as a function of:*
   - *Total mass of the weights at a fixed difference between the masses*
   - *Difference between the masses at a fixed total mass of the weights.*

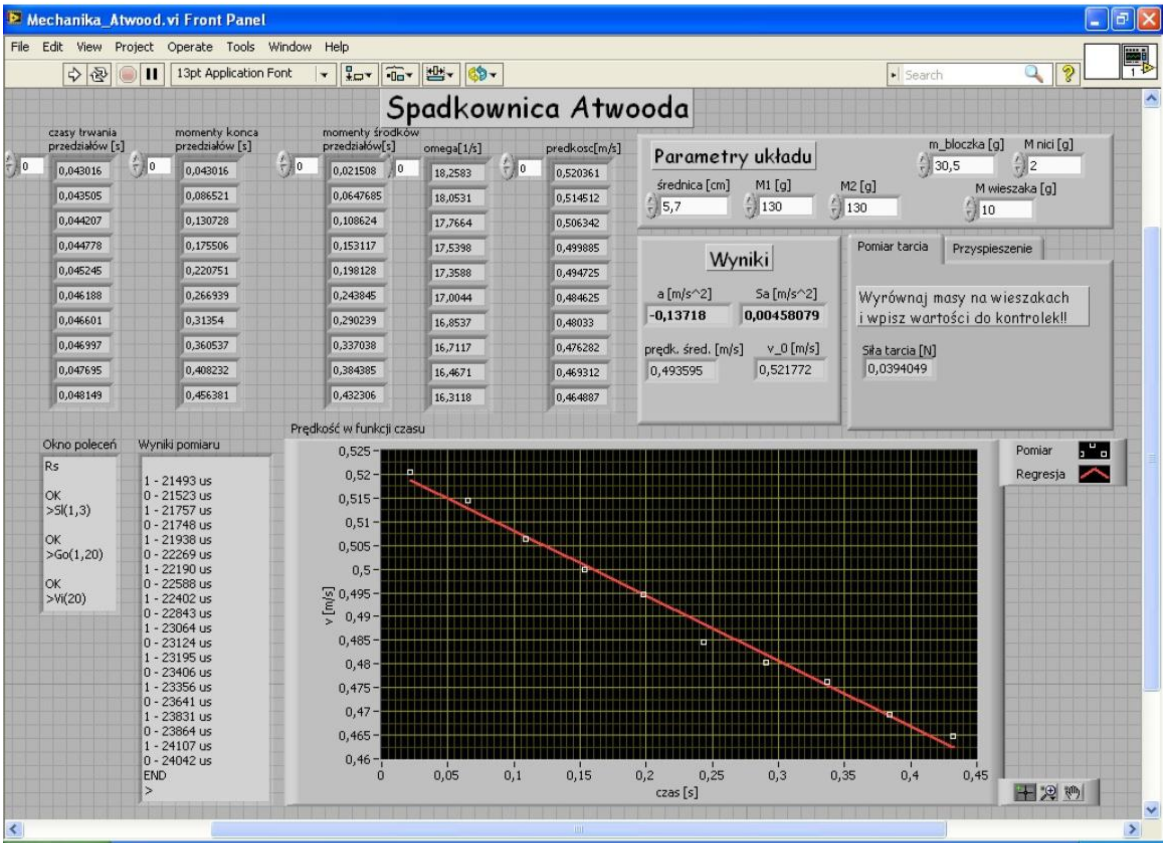
6. *Using eq. (5) calculate the regression coefficients and use them to find the friction force and gravitational constant for both series of measurements. Compare the results with those obtained realising task 3.*

Additional data:

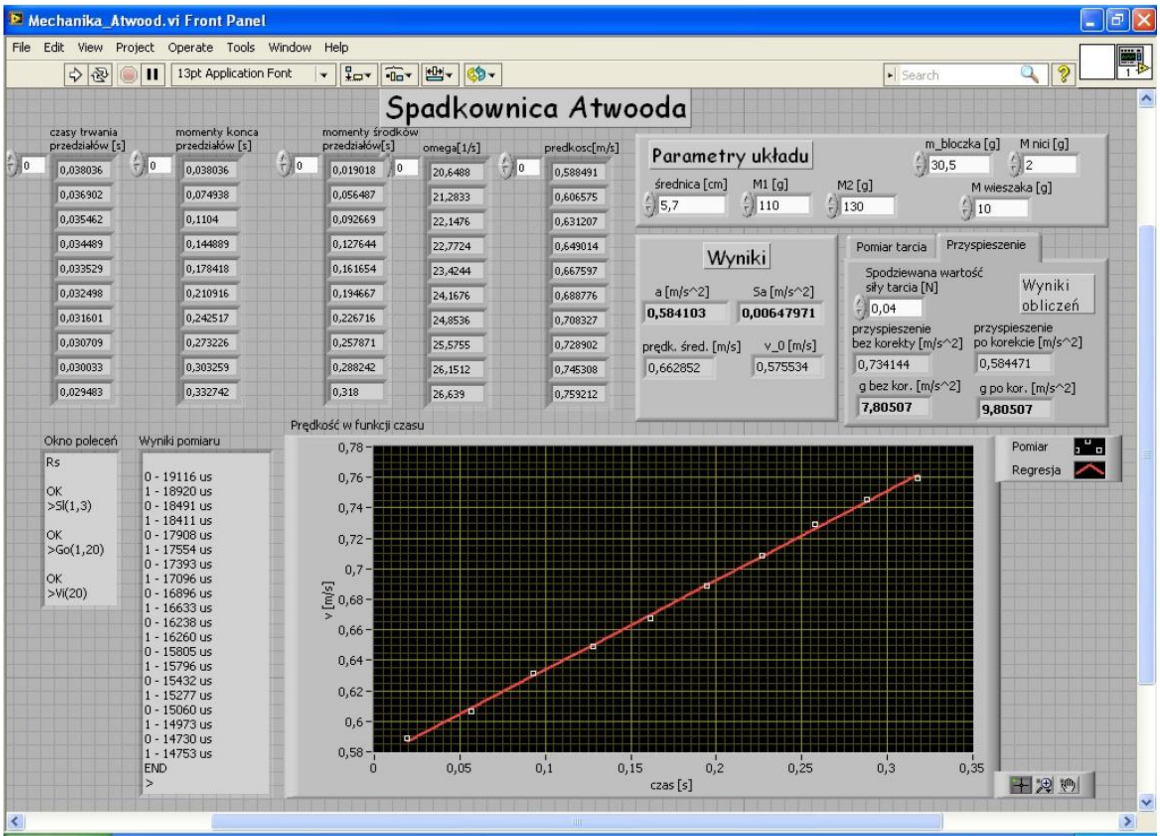
Pulley mass: 30 g

Hook mass: 10g

Thread mass: 2 g



Interface of the measuring program:measurement of friction force



Interface of the measuring program: measurement of acceleration