

D102. Damped Mechanical Oscillator

Aim: design and writing an application for investigation of a damped mechanical oscillator

- *Measurements of free oscillations of a damped oscillator*
- *Measurements of forced oscillations (resonance)*
- *Determination and description of the resonance curve of a damped oscillator*
- *Determination of the Q factor of the resonator*

1. Introduction.

The phenomenon of mechanical resonance is observed in the system in which forced oscillations of a damped oscillator take place. If the oscillator considered is a physical pendulum, then it is subjected to a moment of force, M , proportional to the amplitude (width of the pendulum swing) φ from the position of equilibrium (for small φ ; $\sin(\varphi) = \varphi$) and acting in the opposite direction:

$$M = -D\varphi, \quad (1)$$

where $D=mgd$ is the directing moment of the pendulum (d is the distance from the centre of gravity of the mass m to the axis of rotation of the pendulum). According to the definition of the moment of force we have $M = I\alpha$ (I is the moment of inertia of the pendulum, while the angular acceleration is the second derivative of amplitude over time $\alpha = d^2\varphi/dt^2$). Taking into account these relations we get the differential form of the pendulum equation of motion:

$$\frac{d^2\varphi}{dt^2} = -\frac{D}{I}\varphi \quad (2)$$

Equation (2) describes the swings of pendulum without damping of its movement due to air drag and friction. The solution to equation (2) is:

$$\varphi = \varphi_0 \cos(\omega_0 t + \delta) \quad (3)$$

where φ_0 is the initial amplitude, δ is the initial phase of the swing, $\omega_0 = \sqrt{D/I}$ is the eigen-frequency (natural) of oscillations of the pendulum.

The real pendulum performs damped oscillations whose amplitude decreases in time. The causes of dumping are resistance of the medium and friction. In order to take into regard the effect of friction on the pendulum motion, we have to add to the right side of equation (2) an

additional moment of force, M_F , proportional to the speed of pendulum motion. If the resistance of the medium is proportional to the speed ($M_F = -Hd\varphi/dt$; H is the viscosity coefficient) then the equation of motion takes the form:

$$\frac{d^2\varphi}{dt^2} = -\frac{D}{I}\varphi - \frac{H}{I}\frac{d\varphi}{dt} \quad (4)$$

The solution to this equation is:

$$\varphi = A\cos(\omega_d t + \delta) \quad (5)$$

where

$$A = \varphi_0 \exp(-\beta t) \quad (6)$$

is the decaying in time amplitude of oscillations described by the envelope of damped oscillations. The constant of decay, known as the damping coefficient is given by the relation $2\beta = H/I$. The frequency of damped pendulum ω_d , differs from that of free oscillations.

$$\omega_d^2 = \omega_0^2 - \beta^2 \quad (7)$$

To obtain undamped oscillations, of a constant amplitude, the loss of energy should be continuously compensated for. It is achieved by application of an additional periodically changing moment of force $M_W = M_0 \cos(\omega t)$, M_0 being the moment of driving force. Then the equation of motion becomes

$$\frac{d^2\varphi}{dt^2} = -\frac{D}{I}\varphi - \frac{H}{I}\frac{d\varphi}{dt} + \frac{M_0}{I}\cos(\omega t) \quad (8)$$

and it can be also expressed in the form characteristic of a forced damped oscillator (the symbols are as above):

$$\frac{d^2\varphi}{dt^2} + 2\beta\frac{d\varphi}{dt} + \omega_0^2\varphi = P\cos(\omega t) \quad (9)$$

where $P = M_0/I$.

The solution to equation (8) or (9) has the same form as equation (5), where

$$\delta = \arctan\left(\frac{2\beta\omega}{\omega^2 - \omega_0^2}\right) \quad (10)$$

$$A = \frac{M_0}{I} \frac{1}{\sqrt{(\omega^2 - \omega_0^2)^2 + 4\beta^2\omega^2}} \quad (11)$$

The amplitude of oscillations, A , depends on the amplitude of the moment of driving force, M_0 , frequency of free oscillations, ω_0 , frequency of the forcing moment, ω , and the damping constant β . The highest amplitude, A_{rez} , is observed when the resonance condition is met, so when the frequency of forcing oscillations or driving frequency reaches the value:

$$\omega_{rez}^2 = \omega_0^2 - 2\beta^2 \quad (12)$$

The curve describing the changes in amplitude as a function of frequency of forcing oscillations, $A(\omega)$, is called the curve of mechanical resonance. The width of the resonance curve depends on damping and provides information about the energy loss in the oscillator. The so called Q factor can be defined as the mean energy accumulated in the oscillating system to the mean work done by the force inducing the oscillations per a single period of oscillation. The Q factor can be determined from the parameters of the resonance curve:

$$Q = \frac{\omega_0}{2\Delta\omega_{1/2}} \quad (13)$$

where $\Delta\omega_{1/2}$ is the full width at half maximum (FWHM) of the resonance curve presented as $A^2(\omega)$. When analysing the curve $A(\omega)$, FWHM must be measured not at half maximum but at a level $1/\sqrt{2} \approx 0.71$ of the curve maximum.

2. Equipment.

The damped oscillator studied is a physical pendulum, see Fig. 1. Damping is introduced by the phenomenon of magnetic hindering. At the bottom end of the pendulum an aluminium plate is mounted. Near the plate a handle with strong permanent magnets is placed. When the pendulum swings, the magnetic field produced by the magnets crosses the aluminium plate (a conductor) and induces in it eddy currents (Faraday effect). The eddy currents generate the secondary magnetic field directed in the opposite to the primary field (Lenz rule). These two magnetic fields counteract the motion of the pendulum working as a damper. According to the Faraday's law of induction, the intensity of induced eddy currents (and so the intensity of secondary magnetic field) is proportional to the speed of the pendulum and the attached conducting plate. This method of introducing damping ensures that the force of damping is proportional to the speed of the pendulum, similarly to its motion in viscous environment. The strength of damping force (viscosity coefficient) can be adjusted by changing the distance between the magnets and the aluminium plate. The amplitude of the pendulum swing, that is the angular distance, is measured by an angular position sensor whose operation is based on the so-called Hall effect. The driving force is applied through the springs between which the rod of the pendulum is mounted. The oscillations are transmitted to the pendulum by one of the springs. The source of the driving force is a stepper motor with an eccentric wheel mounted on its axis. The stepped motor is

controlled by a multifunctional measuring card NI USB-6009 which is also used for data collection from the angle meter and their processing. The program for control of the stepper meter, collecting and analysing data is to be written in NI LabVIEW environment.

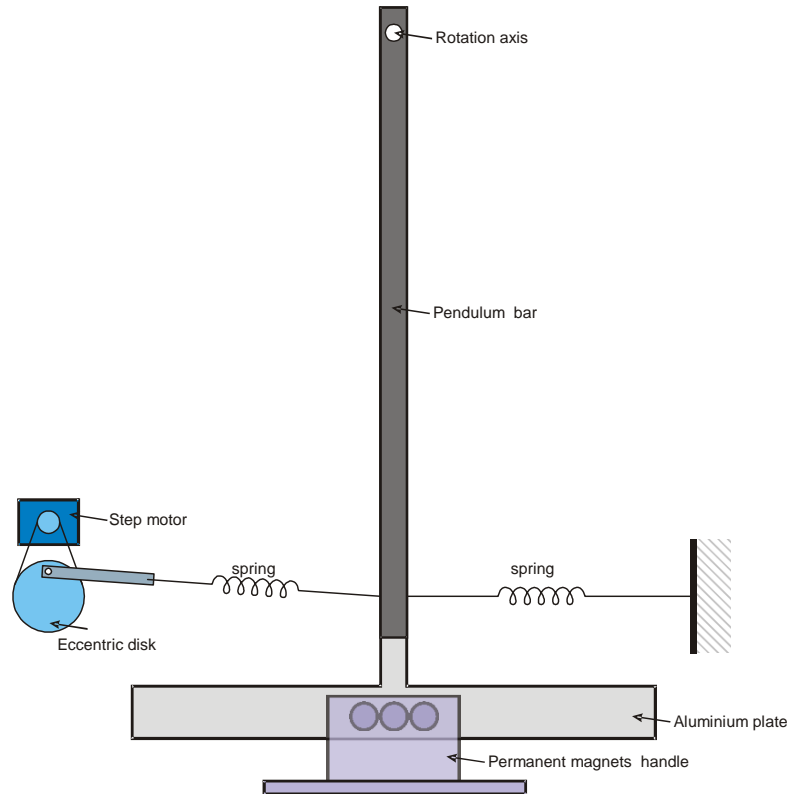


Figure 1.

3. Experiment.

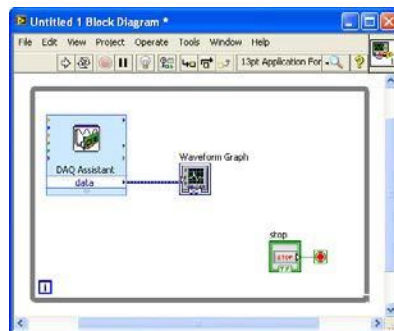
The aim of the experiment is to determine the curve of mechanical resonance, so the dependence of the amplitude of pendulum swing as a function of frequency of the driving force. The program should record time changes in the pendulum amplitude (angle) and should control the frequency of the inducing force. The amplitude of the force strength depends on the construction of the pendulum (elasticity constants of the coupling springs and the amplitude of the eccentric oscillations) and it is constant. Prior to writing the program, the measuring card should be plugged into the USB socket in the computer. Switch on the power supply to the angular position sensor. ***At this stage do not switch on power supply to the stepping meter. READ ON!!!***

Read off the amplitude of pendulum swing

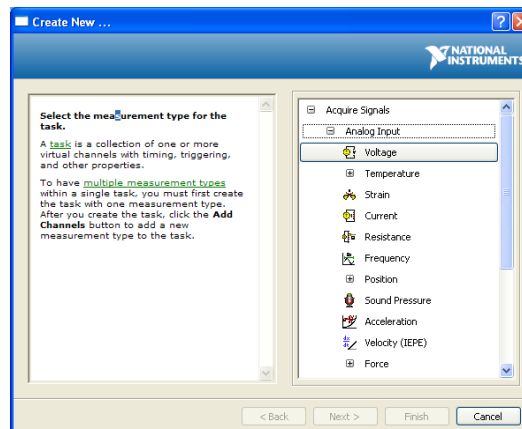
The angle of pendulum swing is measured by a halotrone which is a sensor of

position based on the Hall effect. The sensor provides voltage proportional to the pendulum amplitude. The voltage is read with the help of card NI USB-6009.

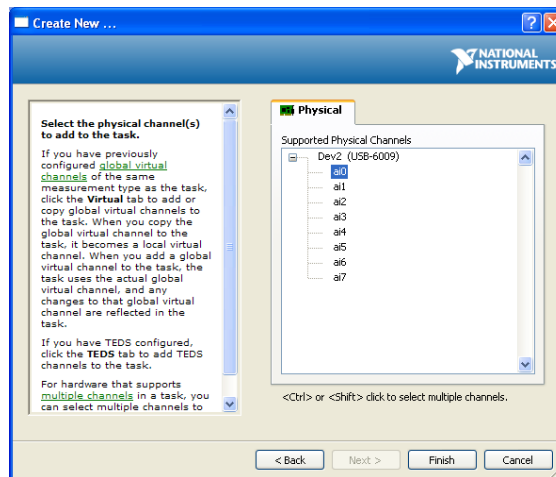
- 1) Check if the electric wires from the halotrone are plugged into AI0 and AI4 pins on card NI USB-6009.
- 2) Record the time course of voltage at a selected analog input pin with the help of *DAQAssistant* in the *WHILE* loop. Display the data packets in the *Waveform Graph*



- a) Record the analog voltage signal



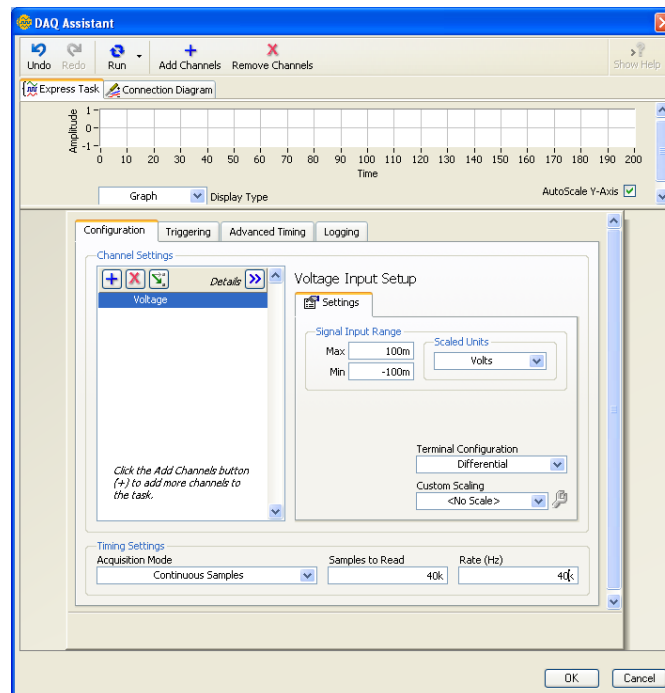
- b) Choose the measuring channel (AI0)



The sensitivity of NI USB-6009 is sufficient for recording of the induced SEM signal with no additional amplification. The induced voltage is expected to be at a level lower than 1V, so the default measuring range of the card should be reduced from +/-10V to +/-1V. The signal measured can be disturbed by outside sources (e.g. from the supply network), so to minimize them, the measurements should be made in the differential mode.

In the following part of the program, use the function *extract single tone information.vi* to determine the amplitude and frequency of oscillations of the sinusoidal signal. This function demands that the signal analysed has a few periods of oscillations. Taking into account that the period of the pendulum is about 1 second, it is enough to analyse a signal of the length of about 5 seconds. This information will be needed for setting the rate of sampling (*Rate*) and the number of samples in the creator *DAQAssistant*. At the sampling rate of 1000 samples per second, it is sufficient to read 5000 samples.

c) Set the conditions of measurements: Signal input range: 1+/-V; Acquisition mode: Continuous Samples; Terminal configuration: Differential; Rate: 1000 S/s; Samples to Read: 5000S.



3) Check the work of the program setting the pendulum in free oscillations by gentle push. The program should provide 5 second long time dependencies of the signal from the Hall sensor.

The signal collected directly from the Hall sensor is loaded with noise. Prior to further analysis The high-frequency noise should be eliminated leaving only the slow-changing signal of the pendulum.

4) Send the signal acquired in DAQ Assistant to the function executing filtration: Filter.

The type of filter should be set to *Low-Pass*. The cut-off frequency should be set to about 100Hz. Check to work of the program.

The filtered signal can be subjected to analysis in order to determine the frequency (f) and amplitude(A)of the pendulum oscillations.

5) The filtered signal from *Filter* send to *Extract single tone information.vi*. Read the Amplitude and Frequency of the signal by creating indicators on proper terminals of the Extract... function.

In order to get the resonance curve for the system studied, create an array of (Frequency,

Amplitude) pairs and display them on the XYGraph indicator.

Control of driving frequency through motor's speed of rotations

The oscillations of selected frequency are forced by the intermediacy of the eccentric system mounted at the axis of the motor. This arrangement ensures the transformation of the rotations of the motor shaft into the oscillations in the plane of the pendulum oscillations. The eccentric system is mounted at the axis of the stepping motor whose rotation speed can be controlled by the voltage applied to the stepper motor controller. The source of voltage controlling the motor rotations is the measuring card NI USB-6009.

6) Put the second copy of *DAQAssistant* inside the *WHILE* loop. Proceed as described at 2) configure the card as a source of an analog signal (*AnalogOutput*). Select AO0 as the output channel (check the connections of wires to card NI). The choice of *Acquisition mode* at *1 Sample On Demand* permits setting a single and constant voltage at AO0. Connect the numeric control to the terminal *Data* of *DAQ Assistant*, the control will enable introduction of the selected value of controlling voltage.

CAUTION!!!For the motor to be still, the control voltage should be exactly 2.5V. Higher or lower voltage change the direction of the motor rotation. In particular the control voltage of 0V (so no voltage at all) corresponds to the maximum rotation speed of the motor. That is why, to avoid the useless work of the motor, power should be supplied to the motor drivers only after configuration of the output channel of the measuring card and selection of the control voltage of 2.5V.

7) Change the control voltage in the range from 0.2V to 1.4V. In order to draw the resonance curve, for each value of control voltage (frequency of driving force) measure the couple (*Frequency, Amplitude*). CAUTION: After each change in the frequency of driving force, the system for a moment is in the transient state. To determine correctly the amplitude of these oscillations, it must not change in time. It is recommended to draw the momentary values of the oscillation amplitude in the *Waveform Chart* and record the values of (*Frequency, Amplitude*) only when the oscillation amplitude does not change any more.

8) Draw the measured resonance curve $A(f)$. In the same coordinates draw the dependence

calculated from the modified equation (11)

$$A(f) = \frac{A_0}{\sqrt{(f_0^2 - f)^2 + 4\beta^2 f^2}}$$

Find the values A_0 , f_0 and β which permit the best fit to experimental data.

9) Determine the Q -factor of the resonator from equation (13).

10) Analyse the free oscillations of the damped oscillator. Displace the pendulum from the resting position by some amplitude, let it go and wait till the oscillations stop as a result of damping. Record the time variations of displacement $A(t)$. In order to do this change the acquisition parameters in the function *DAQ Assistant* (increase the number of samples to read so that the time of measurement would be long enough. Try to describe the envelope of recorded damped oscillations with equation (6)

$$A(t) = \varphi_0 \exp(-\beta t)$$

Compare the value of coefficient β with that obtained in the previous task.

11) Compare the analytical form of equation (9) with the equation describing the phenomenon in electric circuits containing resistance (R), induction(L)and capacity (C) elements (see the description of experiment “The phenomenon of resonance in alternate current circuits”- equation (9)). Analyze the two equations, find analogies, give other phenomena that can be described by equation (9).